Journal of Policy Research, 9(3), 36-51.
https://jprpk.com
https://doi.org/10.5281/zenodo.8360939

Wavelet Analysis of CO_2 Emissions' Co-movement: An Investigation of Lead–lag Effect among Emerging Asian Economies

Maryam Aziz[1], Muhammad Zeeshan Shaukat[2], Abdul Aziz Khan Niazi[3], Abdul Basit[4], Tehmina Fiaz Qazi[5]

Abstract

The rapidly increasing emissions of carbon dioxide as the main greenhouse gas have been witnessed world over in last few decades. In similarity to other continents, Asia has been affected by the carbon dioxide gas emissions over the past decades. A number of researches conducted in the past studied various aspects of CO_2 emissions sources in Asia. However, this study aims to investigate the interrelationship of selected Asian countries regarding the CO_2 emissions co-movements with focus on Pakistan during the period of different crises. We used WDI (World Development Indicators) yearly data for each country and compared Pakistan's metric tons per capita CO_2 emissions with those of China, Indonesia, India, Iran, Japan, Singapore, Korea, Malaysia, Sri Lanka and Turkey. Contrasting with the contemporary methods, the data is analyzed through separate wavelet power spectra, wavelet coherence and cross-wavelet transform for the determination of correlation. These models are potentially capable of analyzing the co-movements among the countries with reference to time and frequency spaces. Study being conducted at the country level; the analytical findings may directly benefit the environmental planning. The findings are useful for policy makers, regulators and environmentalists. Further discussions on environmental problems is expected to be triggered by the results of this study.

***Keywords*:** CO_2 emissions, carbon dioxide emissions, wavelet analysis, climate change, emerging economies

1. Introduction

The deteriorating effects of CO_2 emissions on environment have become a major cause of concern for developing countries in Asia (Rahman & Alam, 2022). Extreme weather conditions, frequents floods, droughts, rise in sea levels and depletion of ozone along with other along with other impacts serve as evidence of global climate change. A key contaminant is carbon dioxide gas emissions responsible for climate change across the continents. The reduction of carbon emissions globally is among one of the ways for climate protection. The growth of industrial sector consequent to increase in human population led to excessive carbon dioxide emissions across the world. Industrial revolution first observed by England, later on Western Europe, northern Europe, central Europe, Russia and Japan experienced this growth. But the revolution is not observed by the Asian countries in the same period (Ahmad, Iqbal & Mahmood, 2013; Audi & Ali, 2023). European countries had witnessed a peak in CO_2 emissions during mid- 1970s. During the past decade, Europe managed to counter CO2 being emitted from fuel combustion. From 2000 to 2010, the emissions from CO_2 declined per year by 0.7% which further declined per year by more than 2% from 2011 to 2015. The use of nuclear energy, oil and natural gas as a substitute to coal is thought to have reduced CO_2 emissions. The EU trade policies have also played a part towards reduction of CO_2 emissions (Neves, Marques & Patrício 2020). In relation to other continents, the per capita emissions of CO_2 in Africa are low. But in upcoming 30 years, the rapid industrialization is likely to cause an increase in these emissions (Njoh, 2021).

The prominent growth of CO_2 emissions in Asia can be seen as a consequence of Asia's growing share in purchasing power parity. The future growth in CO_2 emissions can be predicted from this consistent rise in per capita incomes. High pace of economic growth in Asian countries poses a number of challenges such as urbanization and greenhouse gas emissions. Countries in Japan and Asian Pacific Rim such as China, Singapore, Malaysia, Thailand, Japan, South Korea, Sri Lanka, Turkey, India, Iran and Pakistan have make developments in non-agriculture sectors resulting into a remarkable increase in urbanization. Asia is among the continents where the urbanization is projected to be doubled by 2030. The increase in urbanization results in more vehicular emissions. Deforestation as well as the use of hybrid seeds are contributors to inflated CO_2 amounts in the environment (Parkar & Bhatti, 2020). According to Bajpai (2018), China, India and Singapore are the largest economies of the World reason being they secured second, sixth and thirteenth Place respectively. In Pakistan, economic factors including the foreign direct investments, financial developments, trade, innovation and urbanization and energy consumptions are some of the pivotal factors making their contributions towards increasing CO_2 emissions. To ensure the healthy survival of life, environmental problems must be addressed (Abbasi et al., 2020). The paper focuses on finding out carbon dioxide co-emissions in eleven Asia countries i.e. Pakistan, China,

[1] Institute of Business and Management, University of Engineering and Technology, Lahore, Pakistan
[2] Hailey College of Commerce, University of the Punjab, Lahore, Pakistan
[3] Corresponding Author, Department of Management Sciences, University of Engineering and Technology, Lahore, Pakistan
[4] Lahore Institute of Science & Technology, Lahore, Pakistan
[5] Hailey College of Banking and Finance, University of the Punjab, Lahore, Pakistan

India, Thailand, Malaysia, Singapore, Sri Lanka, Iran, Japan, Korea and Turkey. Finding out the co-movements would enable the policy makers to compare the country policies during different regimes and economic crises (Annexure III).
To study the co-movements of CO_2 emissions in Asian countries, the method being applied is time series analysis through wavelet transforms. Time series analysis helps develop understanding of underlying causes of trends or systemic patterns over time. Utilizing this technique, the time series data is decomposed into multiple time-series that is spread over multiple time scales and enables the researchers to evaluate the co-movements at various frequencies. Time-series analysis allows the researchers to interpret the level of inter-dependencies between various time series, thus helps pinpoint the dynamics lying within the time-series (Zeng et al., 2020). Time series analysis helps to build understanding regarding underlying causes of systematic patterns and trends over extended period of time, thus analysists can delve deeper into the reasons for occurrence of such trends. A number of factors are rationale behind using this approach such as, the requirement of non-stationary data as the CO_2 emissions do not remain stationary over time. The research utilizes phase angle statistics in order to test mechanistic models of physical relationship and to attain confidence in causal relationship between corresponding time series (Rhif et al., 2019).
Researches have been conducted uncovering the contributors of CO_2 emissions in Asian countries. Most of the past researches had their focus on finding the interconnection among the causes of CO_2 emissions in Asian countries (Batool et al., 2022; Audi & Ali, 2023). However, there has been a lack of research towards the co-movements of CO_2 emissions in the Asian countries. This study focuses on Asian countries for various reasons; (1) More than half of total population of the world resides in Asia. (2) Asian countries have surpassed the rest of the world in economic development. Neither of the studies till date attempted to examine the co-movements of CO_2 emissions utilizing the Wavelet approach for Asian countries. After the trade agreements between China-Pakistan and China-US, the focus of trade has shifted towards East Asian and South Asian countries. Therefore, the study has its focus on Asian countries keeping Pakistan at its center. Regional proximity is another reason for choosing Asian countries. The wavelet coherence would identify the data in time as well as in the frequency domain and quantify the time-frequency relationship among the variables involved, thus provides novelty to this study. The study is motivated by aforementioned rationality in Asian countries regarding emissions of CO_2 and contributes towards the contemporary literature by capturing time-frequency causal association between selected Asian countries for multiple time scales utilizing wavelet approach. The use of this advanced approach would enable governments, organizations and companies in selected Asian countries to formulate better environmental policies, hence ensure long-term sustainability. The study is aimed: i) to compare Pakistan's performance on controlling CO_2 emissions to other countries (i.e. China, Indonesia, India, Iran, Korea, Japan, Malaysia, Sri Lanka, Singapore, and Turkey) in order to assess what Pakistan has achieved in contrast to these countries, ii) to determine what needs to change in order to perform better, iii) to analyze the interrelationship among the levels of CO_2 emissions of selected Asian economies in order to explicate the patterns of relationship among them during the past sixty years, and iv) to reveal whether there existed any linkage among the levels of CO_2 emissions of Asian economies over the period of time. The study focused specially on Pakistan as against other eminent Asian Economies A multitude of methodological choices are available for time-series analysis such as Correlation coefficient, GARCH models (Nkalu & Edeme 2019), Copula models (Chapon, Ouarda & Hamdi 2023), MODWT (Zhu, Wang & Fan 2014), Fama-French three factor model (Karp & Van Vuuren, 2017). But Wavelet analysis is best suited for this study comparing to other alternatives available. In scientific applications such as in geophysics (Liu, Hsu & Grafarend 2005; Zamani, Samiee and Kirby 2013; Grinsted, Mooree & Jevrejeva, 2004), in astrophysics (Li, Gao & Zhan 2009; Bloomfield et al. 2004; Donner & Thiel 2007; Kelly et al. 2003), medicine (Qassim, et al. 2013; Hassan et al. 2010; Scholkmann & Dommer 2012) and engineering (Daud & Sudirman 2022), wavelet based models are proved to be a dynamic tool. In the fields of economics (Soni, Nandan & Chatnani 2023), finance (Armah, Amewu & Bossman 2022) and geology (Yu, Wang & Wang 2023), this approach is used as well. Over the conventional methods, wavelet analysis has advantage since it can potentially analyze the time series which are non-periodic and non-stationary containing irregularities such as spikes and discontinuity (Crowley, 2007). Moreover, wavelet analysis has been proved to be an exceptional tool to unveil the lead-lag relationship in time-frequency space and identification of direction and degree of co-movements (Grinsted et al., 2004). Particularly, in environment related research diverse variants of wavelet such as wavelet coherence (WCOH), continuous wavelet transform (CWT) and cross-wavelet transformation (XWT) have previously been used to evaluate the co-movements of factors involved in CO_2 emission (Crowley, 2009). The color spectrums represent the time dynamics captured over number of different frequencies. In comparison with wavelet correlation analysis, wavelet coherence (WCOH) is potentially more capable in identification of intense co-movement regions in both time and frequency space. The wavelet transformation approach is inculcated to circumvent limitations of the Fourier transformation where the location information being stored in phases is difficult to extract. The Fourier transformation is, in fact, sensitive to changes in function (Wirsing, 2020). The remaining article is organized as stated hereinafter. In next section, relevant literature is reviewed, while in Section 3, methodology utilized in this study is presented. Sections 4 and 5 discuss the research results and conclusions respectively.

2. Literature Review

To set out the outset of the study, the relevant literature is reviewed. Databases such as Taylor & Francis, Jstor, Emerald, Wiley-Blackwell and Science-Direct are explored utilizing Google as search engine. Keywords used are: CO_2 Emissions, Carbon dioxide emissions, CO_2 emissions in Asia, CO_2 and greenhouse gas emissions, CO_2 emission metric tons per capita, CO_2 emission calculations, Carbon dioxide emission overview, CO_2 emission Asian overview, Co2 and climate change, CO_2 emissions and energy economics, continuous wavelet transform, CO_2 emission from fuel combustion, fossil CO_2 emissions, wavelet, wavelet coherence and cross-wavelet transform. The studies are in abundance regarding CO_2 emission trends in Asian countries but focus of this study is to investigate the co-movements of per capita CO_2 emissions in selected Asian countries.

In the form of climate change and environmental degradation, mounting challenges are being faced all around the globe as grave risks are posed to human lives. Countries are under immense pressure to sustain the well-being of populations (Li & Zhou 2019). A significant growth in globalization as well as urbanization has been witnessed in last two decades. Developing nations have gone through considerable level of urbanization (Salahuddin et al., 2019). 64 % of the population in developing nations is anticipated to be urbanized by 2050 Shahbaz et al., 2016). The nexus between urbanization, CO_2 emissions and economic growth in developing nations has caught the significant attention of the researchers for example (Sun et al., 2018; Faisal et al., 2018).

Over the past two decades, a notable change in global economic power in Asian region is evident. In terms of world income, share of Asia that was 34.4% in 2000 was staggeringly grown to 48.4% in 2017. The CO_2 emissions is consequentially prominent in Asia. Consistent rise in per capita income envisage growing CO_2 emissions from Asian countries in future (Parker & Bhatti 2020). In an effort to fulfill energy needs of the growing population, installation of giant energy projects resulted in tons of CO_2 emissions, posing far reaching impacts. After the trade agreements between China and Pakistan, the execution of coal fired power plants under CPEC, we are on the way to witness a drastic rise by 14,000 metric tons of CO_2 emissions (Khayyam & Nazar, 2021). In emerging Asian economies, CO_2 emissions rise by about 0.22% as consequence of 1% increase in economic growth (Hanif et al., 2019). Understanding the co-movements of CO_2 emissions in Asian countries, pointing out the leading and lagging countries at specific point of times is vital for the policy makers to craft strategies and policies to counter CO_2 emissions (Parker & Bhatti 2020). Environmental policies responded differently in respective Asian countries at specific points in time over the past few decades, therefore there is a need to study their carbon dioxide emissions co-movements at various points in time (Azami & Angazbani, 2020).

There has been burgeoning research related to the CO_2 emissions in Asian countries, attempted to uncover the dynamics and drivers of CO_2 emissions (Parker & Bhatti, 2020), impacts of financial development indicators on CO_2 emissions in East Asia (Ziaei, 2015), impact of urbanization on CO_2 emissions (Anwar, Younis and Ullah, 2020) and relationships between CO_2 emissions, economic growth and urbanization (Khoshnevis Yazdi & Golestani Dariani, 2019). Plentiful literature related to CO_2 emissions employing Fourier based wavelets method is present (Erdogan & Solarin, 2021). The studies covered various continents Mutascu & Sokic (2020), and different regions in Asia (Adebayo & Beton Kalmaz, 2021). But to the best of our knowledge the co-movements of per capita CO_2 emissions in Asian countries have not been studied yet. Disclosing the interrelationship of CO_2 emissions causes has been the focus of most of the studies (Akbar et al., 2020). There is a need for newer studies to look into the problem with a different aspect. With global dynamics changing every minute, we felt a need for study that could disclose the interrelationship of selected Asian countries w.r.t CO_2 emissions over extended period of time. We, therefore, attempted to address the neglected issue while keeping focusing on Pakistan and emerging Asian markets over a period of fifty-six years so that the void in the relevant literature can be filled.

3. Methodology

This study used CO2 emissions data of eleven Asian Countries comprising of Pakistan, China, India, Indonesia, Iran, Malaysia, Singapore, Sri Lanka, Turkey, Korea and Japan over 57 years. The selection of Asian countries for their inclusion in this study depends on the availability of data. We collected yearly data of CO2 emission of these countries starting from 1960 to 2016. Secondary data is collected from data bank of World Development Indicators Website. The sample period considered in this study covers crucial events in which financial crises occurs at different points of time such as 1970s energy crises, 1991 Indian economic crises, Swedish banking crises (1990s), Mexican economic crises (1994), Late 2000s recession worldwide, Russian financial crises (2008-2009) and Spanish financial crises (2008-2009) (Annexure III). In this study, deduction is used as an approach and positivism is the research philosophy.

The research design as shown in Figure 1 comprises of literature review, determining the research gap, secondary data extraction from WDI website, adjustment of data periodicity, generation of time-series plots, separate power spectra, cross-wavelet transform spectra, wavelet-coherence spectra and discussion of results qua reality.

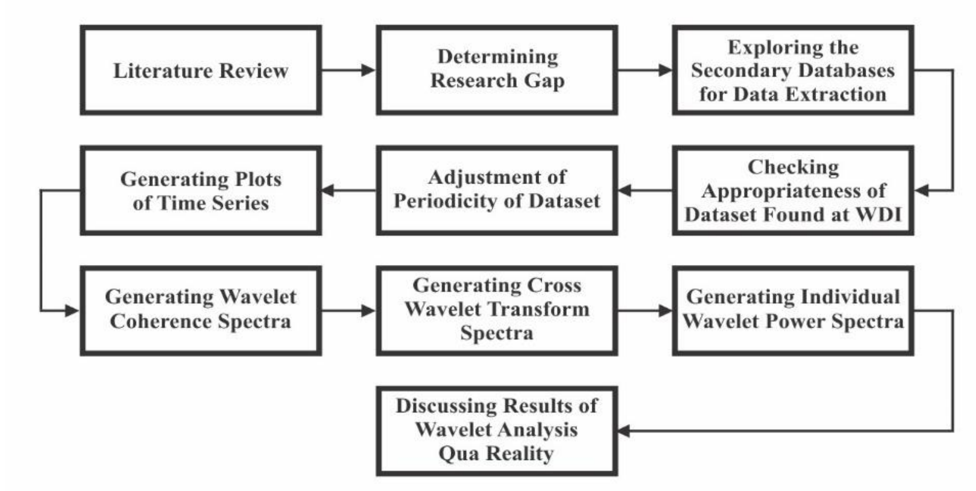


Fig 1: Schema of Methodology (Source: Qazi, Niazi & Ahmad, 2022)

A wavelet approach is used for the determination of interrelationship among the selected Asian countries reason being wavelets possess the capability to capture the non-stationary behavior and time-varying trends. Formal and detailed description of wavelets are provided in various studies (Wu et al., 2022). The suitability of wavelet as a tool for analyzing the Asian countries CO2 emissions co-movements in frequency-time domain is discussed later in the sub-section. In order to examine the relationship between two time-series, cross-wavelet transforms, wavelet coherence and individual power spectrum are discussed in this study. This research, in addition, shows that for the purpose to test mechanistic models of physical relationships and to gain confidence in causal relationships between the time-series, the phase angle statistics can be utilized. For the assessment of statistical significance against red noise, Monte Carlo methods are employed. Moreover, wavelets put forth more accurate timings of shocks by which a change in interrelationship is caused between time-series (Gallegati, 2018).

In this study deduction is used as an approach. Time-series analysis is used for the data that constantly keeps on fluctuating over time. Time series analysis is frequently used in stock market analysis Das & Manoharan (2019), forecasting weather changes Buschow & Friederichs (2020) and heart rate monitoring (Dela Cruz, 2021). Time series analysis helps in identification of the patterns and time series forecasting can predict the future (Sezer, Gudelek & Ozbayoglu, 2020). Conversion of time series data into right time intervals has always been a common problem encountered by researchers as the data is found to be in monthly, quarterly or annual intervals. Moreover, at times data is either in daily or weekly intervals. Therefore, the conversion of data into same frequency is researcher's first task reason being consistent time-intervals are required in most of the time series analysis.

Yearly data of Asian countries CO2 emission was available at WDI Website, the periodicity of which is adjusted using software EViews.

3.1. The Wavelet

Localized in time, wavelet is a wave-like oscillation. Scale and location both are the two basic properties associated to wavelets. Scale (or dilation) explains how "stretched" a wavelet is. This property is related to frequency as defined for waves. Location explains the location of wavelet in time (or space). A wavelet is visualized as a "brief oscillation" having an amplitude that starts at zero, increases, and eventually decreases back to zero. Wavelet can divide continuous-time signals into various scale components as it is a mathematical function, brought into use for diving a given function. In this study, for the assessment of Asian countries CO2 emission co-movements, wavelet is utilized as specified by Morlet's due to its adaptability. A time-series ($\psi_u(t)$) is decomposed into components form with help of wavelet. We used the notations as used by Abban et al. (2022), given in Annexure I and defined wavelet as:

$$\psi|t| = \frac{1}{\sqrt{s}}\psi\left|\frac{t-u}{s}\right|$$

Defined over the real axis, it must be understood that wavelet is real-valued or a complex-valued function $\psi(.)$. It is an assumption that $\psi(.) \in L^2(\mathbb{R})$, which means that wavelet is square-integrable function. The parameter for dilation of scale is represented by s, the position of respective wavelet is represented by u. factor of normalization is shown by $1/\sqrt{s}$ ensuring $\|\psi_{u,s}\|^2 = 1$. We defined wavelet which is specified by Morlet as:

$$\psi_0^M|t| = \pi^{-1/4}e^{i\omega_0 t}e^{\frac{-t^2}{2}}$$

Here**,** wavelet's central frequency is represented by $\boldsymbol{\omega_\circ}$.

3.2. Continuous wavelets

Abban et al. (2022) described the continuous wavelet transforms as:

$$W_x|u,s| = \int_{-\infty}^{\infty} x(t)\frac{1}{\sqrt{s}}\overline{\psi \mid \frac{t-u}{s} \mid dt}$$

ψ(.) represents specific wavelet and its projection is utilized to find $W_x(u,s)$. The cross wavelet transform has an ability to decompose as well as combine $x(t) \in L^2(\mathbb{R})$ such that:

$$x|t| = \frac{1}{c_\psi}\int_0^{\infty}\left[\int_{-\infty}^{\infty} W_x(u,s)\psi_{u,s}(t)d_u\right]\frac{d_s}{S^2}, s > 0$$

The power spectrum variance is specified as:

$$x|t| = \frac{1}{c_\psi}\int_0^{\infty}\left[\int_{-\infty}^{\infty} |W_x(u,s)^2| d_u\right]\frac{d_s}{S^2}$$

3.3. Wavelet power spectrum

Wavelet power spectrum can be defined as $|W_{n,x}|^2$ and local variance of variables can be evaluated through it. As used in Luo et al. (2022), Monte Carlo simulations can be utilized at space s instance and time n in order to get respective local wavelet power spectrum distribution such as follows:

$$D\left(\frac{|W_n^x(s)^2|}{\sigma^2 x} < p\right) \Rightarrow \frac{1}{2}P_f \mathcal{X}^2 v$$

Where v has values 1 or 2 for complex or real wavelets whereas, at the Fourier frequency *f*, P_f is the mean of spectrum.

3.4. Cross-wavelet transform, wavelet coherence and phase differences

Cross-Wavelet Power (XWP) preserves neighborhood covariance of two-period series in all of the frequencies. In this research, XWP is employed to locate the high CO2 emissions co-movement regions in time-frequency domain. The two signal cross-wavelet can be defined as follows

$$W_n^{XY}(s) = W_n^X(s)W_n^{Y^*}(s)$$

Complex conjugate of $W_n^X(s)$ is denoted by $W_n^{X^*}$. The cross-wavelet power's theoretical distribution of two signals with P_K^X and P_K^Y as power spectra is given as follows:

$$D\left(\frac{\left|W_n^X(s)W_n^{Y^*}(s)\right|}{\sigma_X\sigma_Y} < p\right) = \frac{Z_v(p)}{v}\sqrt{P_{\mathcal{K}}^X P_{\mathcal{K}}^Y}$$

Standard deviations of x and y is denoted by $\sigma_X \, and \, \sigma_Y$, the confidence interval is represented by $Z_v(p)$, where P is probability for probability density function. Younis et al. (2020) computed wavelet coherence as:

$$R^2(u,s) = \frac{\left|S\left(s^{-1}W_{xy}(u,s)\right)\right|^2}{S(s^{-1}|W_x(u,s)|^2)\cdot S\left(s^{-1}\left|W_y(u,s)\right|^2\right)}$$

Where S represents the smoothing parameter. Inequality condition of $0 \le R^2(u,s) \le 1$ is satisfied by the coefficient of squared wavelet-coherence (CSWC). If $R^2(u,s)$ approaches to zero, the correlation is considered as weak. Above-mentioned are the reasons behind the suitability of wavelet coherence approach for inspection of variables w.r.t. to time-frequency. For the purpose of distinguishing between the phase relationship, the two time-series phase-difference variables i.e. $\phi_{x,y}$ are utilized. Defined below is the phase difference used for the determination of positions in pseudo-cycle.

$$\phi_{x,y} = \tan^{-1}\left(\frac{\mathfrak{I}\{W_n^{xy}\}}{\mathcal{R}\{W_n^{xy}\}}\right) \text{ with } \phi_{x,y} \in [-\pi,\pi]$$

The phase relationship is characterized by the direction of arrows. The arrows directed towards rights suggest that the variables are in phase and positive correlation exists between them. If the direction of arrows is towards left, the variables are anti-phase and negatively correlated. Moreover, if the arrows move right and up, then the variables are positively correlated and first variable x is leading, whereas if the arrows approach left and up, first variable x is lagging and their correlation is negative. In case, the arrows are right and down, positive correlation is found to be existed between the variables and x is the lagging variable. The correlation is negative and the first variable is leading when the arrows are left and down.

4. Analysis, Results and Discussion

4.1. Analysis

Table 1 shows the CO2 emissions data of eleven Asian countries collected from World Development Indicators from year 1960 to 2016.

Table 1: CO2 Emissions (metric tons per capita)

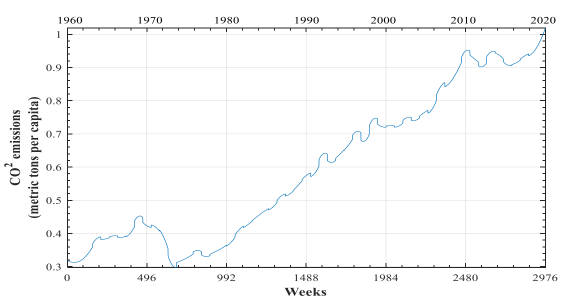
Year	China	Indonesia	India	Iran	Malaysia	Pakistan	Singapore	Sri Lanka	Turkey	Korea	Japan
1960	1.1704	0.2439	0.2676	1.7069	0.4374	0.3146	0.8464	0.2288	0.6123	0.5018	2.5165
1961	0.8360	0.2888	0.2837	1.6255	0.4753	0.3182	1.2299	0.2310	0.6169	0.5613	2.9820
1962	0.6614	0.2486	0.3059	1.6238	0.4687	0.3412	1.4729	0.2476	0.7502	0.6517	3.0597
-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-
1988	2.1509	0.7553	0.6300	3.3181	2.5099	0.5742	12.6858	0.2068	2.4265	5.2808	8.0667
1989	2.1531	0.7349	0.6769	3.4877	2.8457	0.5832	14.2919	0.2036	2.6290	5.5560	8.3299
-	-	-	-	-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-	-	-	-	-
2014	7.5439	1.4500	1.7233	8.4217	8.1252	0.9340	10.3559	0.8627	4.4788	11.5703	9.4808
2015	7.3985	1.9845	1.7843	8.2821	7.7574	0.9454	11.1024	0.9592	4.4612	11.7094	9.1536
2016	7.1758	2.1538	1.8178	8.3167	8.0916	0.9878	6.6940	1.1018	4.6695	12.1111	8.9444

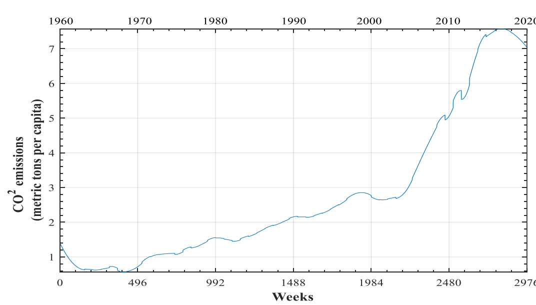
In order to expound the data set used for wavelet analysis in the study, classical descriptive statistical analysis is represented in table 2.

Table 2: Descriptive Statistics of Asian Countries

	China	India	Indonesia	Iran	Japan	Korea	Malaysia	Pakistan	Singapore	Sri-Lanka	Turkey
Mean	2.734324	0.759772	0.942992	4.613772	7.91495	5.838647	3.780859	0.598191	9.6839885	0.395952	2.497876
Standard Error	0.038619	0.007953	0.010545	0.036285	0.036536	0.070729	0.047345	0.004062	0.0825644	0.004011	0.021393
Median	2.141152	0.629455	0.738809	4.232446	8.387705	5.296846	2.585161	0.572682	10.907337	0.281373	2.548665
Standard Deviation	2.106397	0.43376	0.575148	1.979093	1.992781	3.857787	2.582352	0.221552	4.5033565	0.218787	1.166867
Sample Variance	4.436907	0.188148	0.330795	3.916811	3.971176	14.88252	6.668543	0.049085	20.28022	0.047868	1.361579
Kurtosis	0.141216	-0.23811	-0.91199	-0.85508	0.905432	-1.46074	-1.39799	-1.36811	-0.5420796	0.881375	-1.08714
Skewness	1.164513	0.886843	0.469376	0.510784	-1.39924	0.153915	0.340632	0.286779	-0.6737487	1.280573	0.146682
Range	7.016499	1.563479	2.004034	6.845833	7.736102	11.91299	8.084553	0.720748	18.327582	0.989659	4.254039
Minimum	0.558938	0.261698	0.193619	1.604025	2.158992	0.482755	0.404011	0.298183	0.2555646	0.197581	0.592783
Maximum	7.575436	1.825177	2.197653	8.449858	9.895093	12.39575	8.488564	1.018932	18.583146	1.18724	4.846821
Sum	8134.614	2260.322	2805.402	13725.97	23546.98	17369.97	11248.05	1779.618	28809.866	1177.956	7431.18
Count	2975	2975	2975	2975	2975	2975	2975	2975	2975	2975	2975
Largest(1)	7.575436	1.825177	2.197653	8.449858	9.895093	12.39575	8.488564	1.018932	18.583146	1.18724	4.846821
Smallest(1)	0.558938	0.261698	0.193619	1.604025	2.158992	0.482755	0.404011	0.298183	0.2555646	0.197581	0.592783
Confidence Level (95%)	0.075722	0.015593	0.020676	0.071146	0.071638	0.138682	0.092832	0.007964	0.1618891	0.007865	0.041947

The lowest and highest standard deviation is found to be 0.218787 and 4.5033565 for Sri Lanka and Singapore respectively. CO_2 emissions for ten out of eleven Asian countries is positively skewed. Japan is the only country for which the skewness is found to be negative. The kurtosis for all of the eleven Asian countries is found to be less than 3 indicating that the data is not distributed normally for any of the countries.





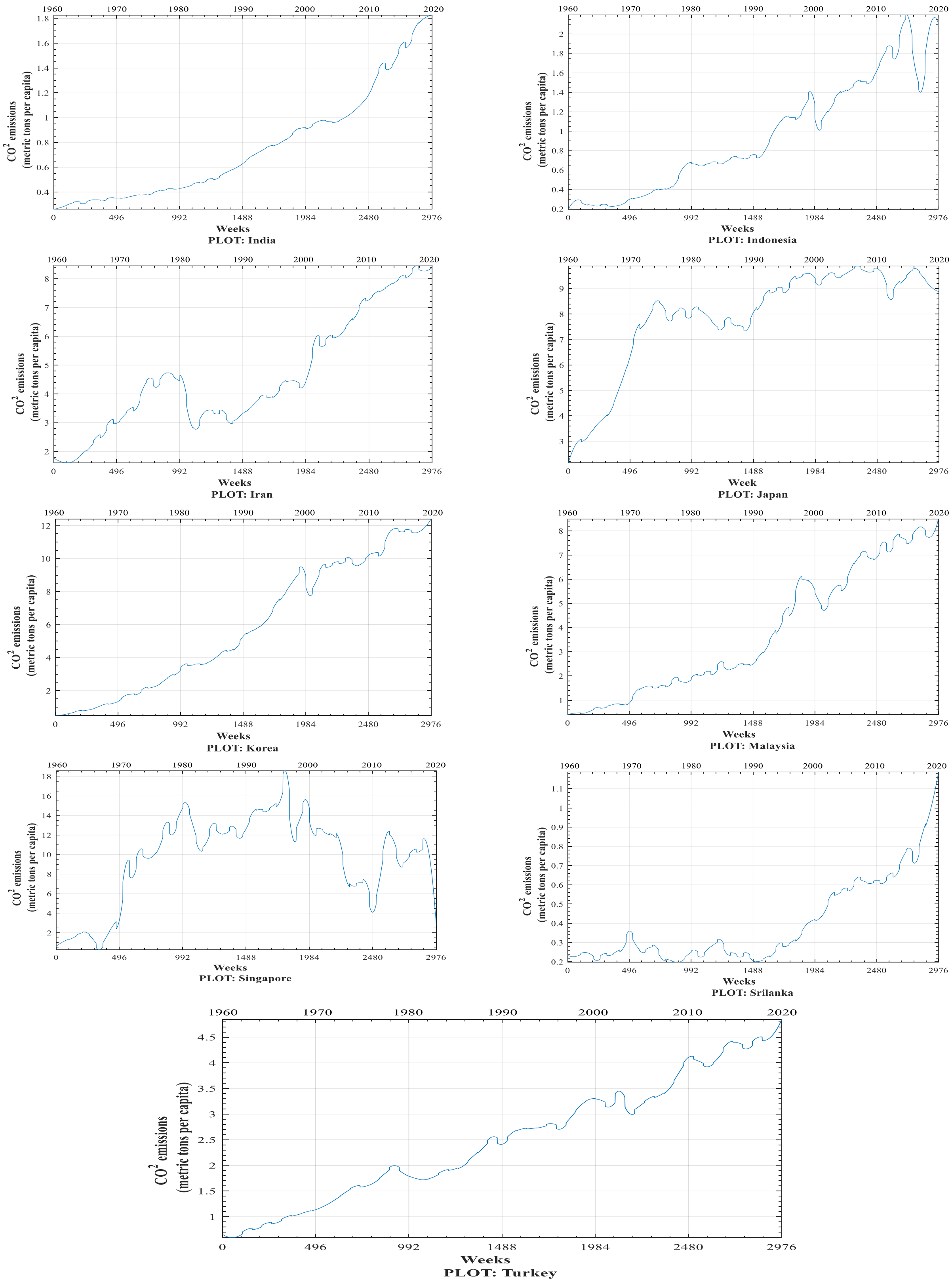


Fig 2: Time-Series Graphs

Fig 2. Shows the time-series graphs of eleven Asian Countries. The data is collected from year 1960 to 2016. The yearly data after conversion into weeks is plotted with an interval of 496 weeks. Co2 emissions (metric tons per capita) are plotted against time. CO2 emissions increasing trends can be clearly seen for the Asian countries except for Singapore where the sharp fall can be noticed after 2010.

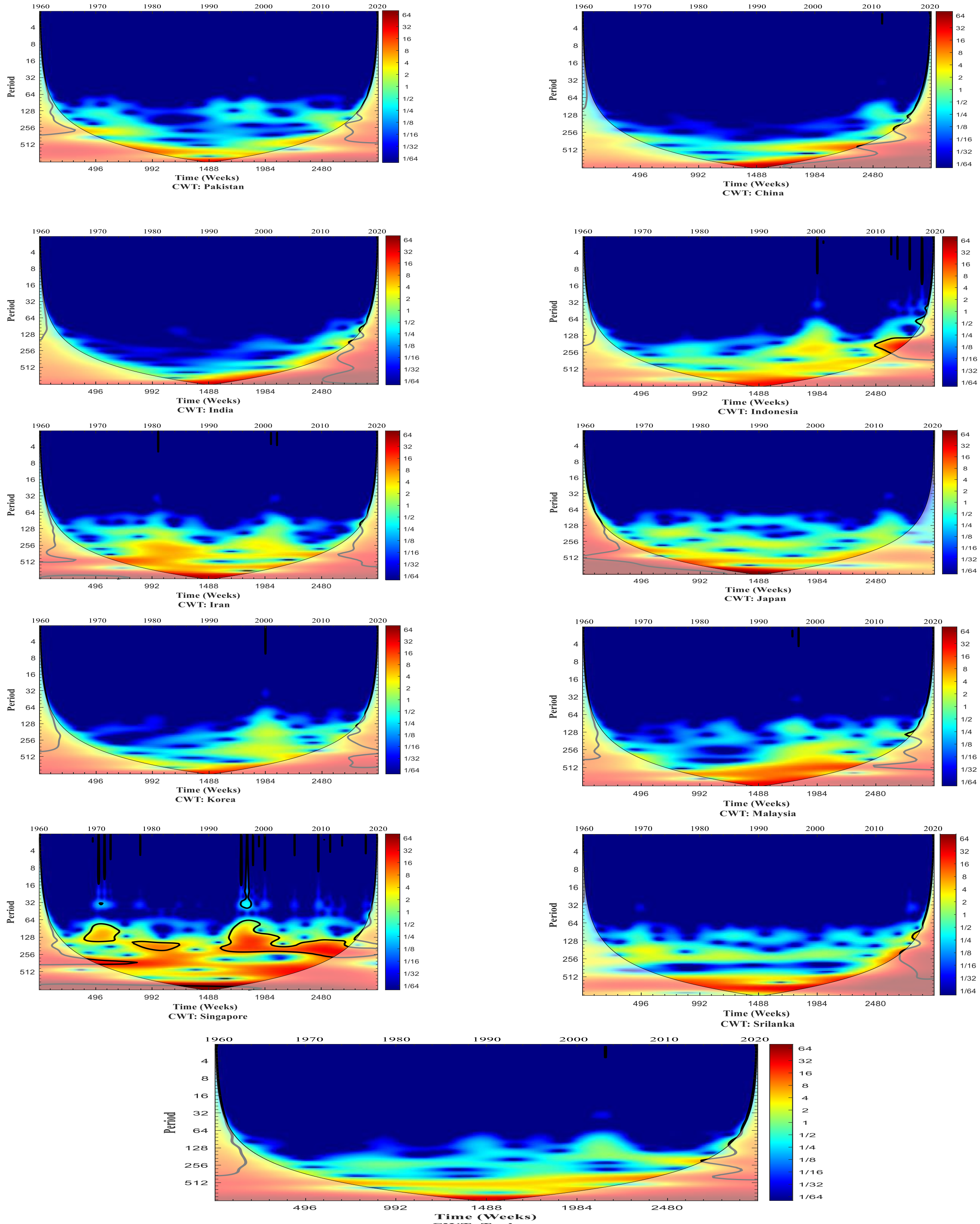


Fig 3: Continuous Wavelet Transform of Asian Countries CO2 emissions

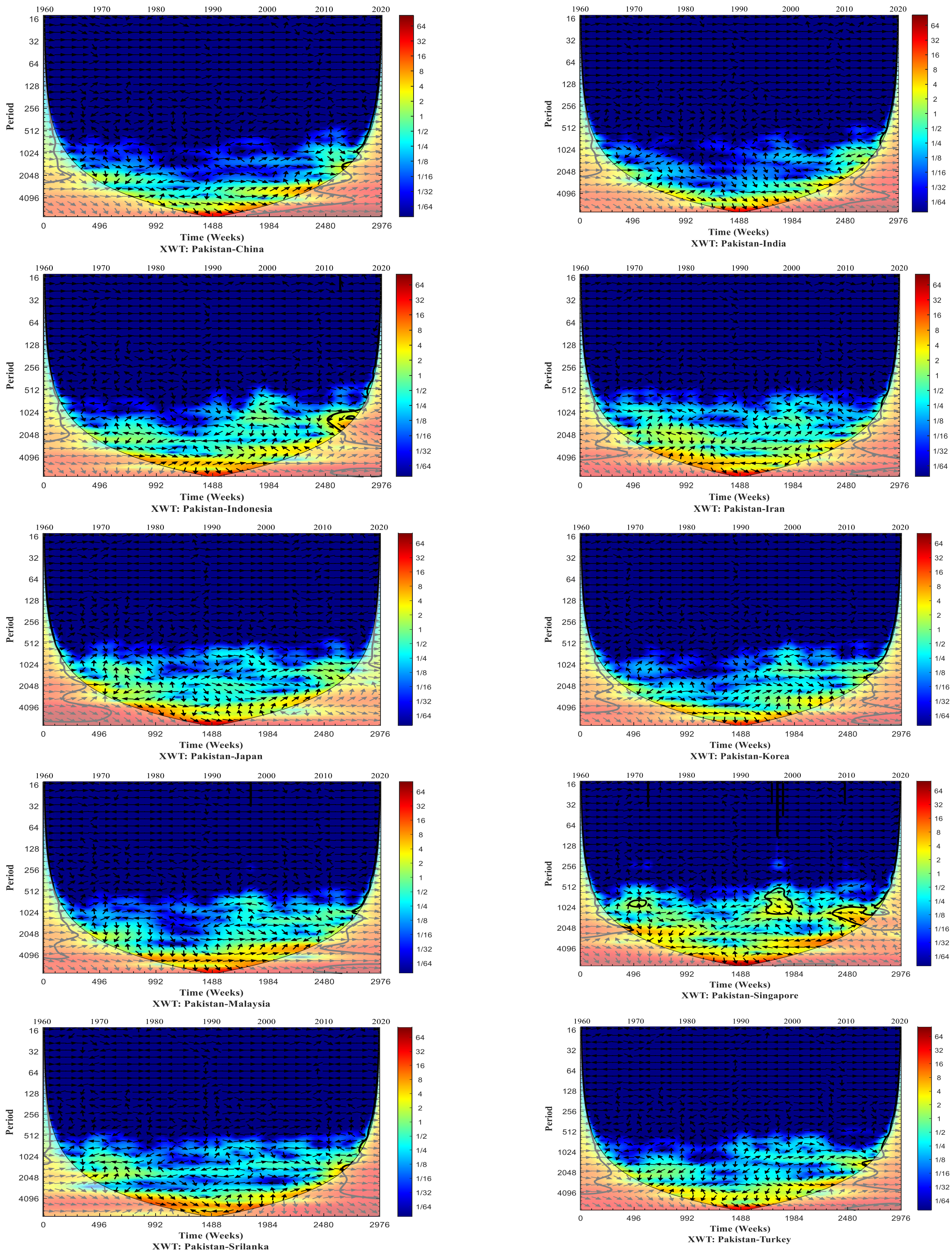


Fig 4: Cross-Wavelet Power Spectra of Asian Countries CO2 Emissions

Fig 3 represent the results of continuous wavelet transform. A contour plot of wavelet power spectrum in three dimensions has been utilized in the study. Frequency component is displayed y-axis and time component on x-axis. Color-map jet is used ranging from dark blue (low power) to dark red (high power). Down the vertical axis, intensity variations can be read over the wavelet scale (keeping time scale constant) whereas across the horizontal axis, intensity variations over the time scale can be read (keeping wavelet scale constant). With the help of phase randomized surrogate series, this is achieved through Monte Carlo simulations. Regions of 5% cumulative area wise significance is enclosed within thick black region where higher intensity spectra indicate the higher variance. This region is termed as COI (cone of influence) where there is unreliability in the attained results. The COI in its lighter shade separates lower intensity regions from those of higher intensity regions.

The white contour designates significance level (i.e. 5%) against a white noise null. The black contour designates significance level (i.e. 5%) estimated from Monte Carlo simulations using phase-randomized-surrogate-series. The COI (i.e. Cone Of Influence) that indicates regions affected by edge effects, is in form of dotted-line. The color code for power ranges from blue (low or cool power) to red (high or warmer power)

Fig 4. Represents cross-wavelet power spectra of Asian Countries CO2 Emissions. The cross-wavelet significant at the 5% is enclosed within thick black contour. Lighter shade indicates COI delimiting the important power regions. Arrows indicate the phase difference between the two time-series. The arrows directed towards rights suggest that the variables are in phase and positive correlation exists between them. If the direction of arrows is towards left, the variables are anti-phase and negatively correlated. Moreover, if the arrows move right and up, then the variables are positively correlated and first variable x is leading, whereas if the arrows approach left and up, first variable x is lagging and their correlation is negative. In case, the arrows are right and down, positive correlation is found to be existed between the variables and x is the lagging variable. The correlation is negative and the first variable is leading when the arrows are left and down. Frequency (periods) and time (weeks) are represented on vertical and horizontal axis respectively

Fig 5. Represent wavelet coherence Asian Countries CO2 Emissions. Wavelet-coherence significant at the 5% is enclosed within thick black contour. Lighter shade indicates COI delimiting the important power regions. Phase difference between the two time series is indicated by arrows. The arrows directed towards rights suggest that the variables are in phase and positive correlation exists between them. If the direction of arrows is towards left, the variables are anti-phase and negatively correlated. Moreover, if the arrows move right and up, then the variables are positively correlated and first variable x is leading, whereas if the arrows approach left and up, first variable x is lagging and their correlation is negative. In case, the arrows are right and down, positive correlation is found to be existed between the variables and x is the lagging variable. The correlation is negative and the first variable is leading when the arrows are left and down. Frequency (periods) and time (weeks) are represented on vertical and horizontal axis respectively

4.2. Results and Discussion

Assessment of CO_2 emissions by a country is importance because it has adverse effects on climate. It is the major contributor to global warming. Assessing the position of emerging economies as against its regional counter parts remained high in research agenda. This study is also aimed to assess position of Pakistan as against eminent Asian economies. Using the historical times series data of CO_2 emissions (kg per 2010 US$ of GDP) taken from WDI (Table 1) Pakistan's position is assessed using basic statistical techniques coupled with Wavelet analysis. Descriptive statistics of CO_2 emissions (kg per 2010 US$ of GDP) by aforementioned countries for last fifty six years based on the data (Table 1) reveal that Singapore has highest mean CO_2 emissions i.e. 9.6839885 (kg per 2010 US$ of GDP) as against lowest by Sri Lanka i.e. 0.395952. Singapore is seconded by Japan. The descriptive statistics in this way provides clues to understand the general characteristics of the data interpretation of each descriptive statistic (Table 2) is deliberately skipped as it may drive the readers out of focus. The close observation of 2-dimensional time-series plots that only present one variable (CO_2 emissions kg per 2010 US$ of GDP) of eleven selected countries where x-axis shows time and y-axis shows the values of measured variable reveal that there is clear cut increasing trend in CO_2 emissions by all countries with an exception of Singapore. There is reduction of CO_2 emissions in Singapore since 1997-98. Continuous Wavelet Transform of Asian Countries CO_2 emissions (wavelet power spectra) yield consistent results with the time series plots, reflecting several ups and downs according to the industrialization arenas. The study determines co-movement and lead-lag relationships are highlighted. Therefore, to measure concisely the influences among the countries, wavelet coherency and phase difference is applied to gauge the degree of co-movement among CO_2 emissions Asian Countries CO2 and the lead–lag effect of different countries. Morlet wavelet transform cross coherency among CO_2 emission indices of selected eleven aforementioned Asian countries. The significance values are derived from Monte Carlo simulations, thick black contour is level of significance level (i.e.5%), whereas, outside of thin line is boundary-affected-zone. The x-axis shows time, whereas, the y-axis shows the period (weeks). Wavelet-coherence finds regions localized both in time-frequency-space where two time-series co-vary. Regions with significant-dependence are represented by regions inside black lines. The warmer the color of a region means higher degree of dependence between the pair. Cold regions outside significant areas denotes time and frequencies with no dependence. It can be seen that CO_2 emissions by both frequency and time intervals move together significantly. The scales are set up to 4094. Very interesting results of wavelet coherence can be observed by inspecting the spectra. At a glance the results confirm findings as against the previous analyses. A comparative analysis of the coherence spectra of eleven indices shows coherence i.e. lead–lag relationship is highly complicated.

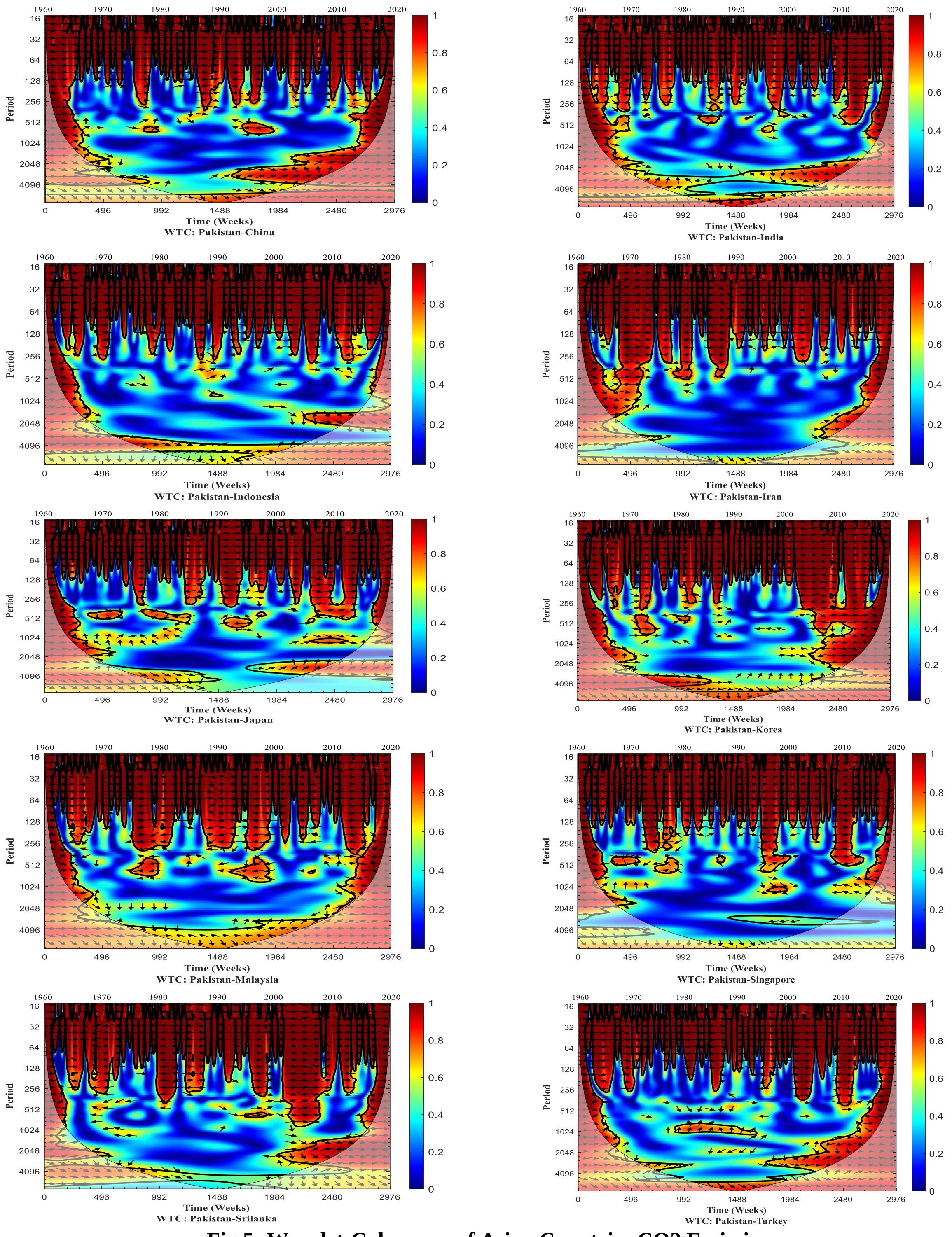


Fig 5: Wavelet-Coherence of Asian Countries CO2 Emissions

This study is also aimed to assess position of Pakistan as against eminent Asian economies. Using the historical times series data of CO_2 emissions (kg per 2010 US$ of GDP) taken from WDI (Table 1) Pakistan's position is assessed using basic statistical techniques coupled with Wavelet analysis. Overall, wavelet power spectra yield consistent results with time plots and previous studies. This study has data set different, methodology different, context different, result different, and countries different. The study has practical as well as theoretical implication as given hereinafter Practical implications for environmental agencies, political governments and policy makers, industry across economies, research community, environmental scientists and society at large because it provides lot of information for understanding the transformation of phenomenon over the period of time. The analysis is based on real time data taken from highly reliable sources and analyzed through the state of the art technique of analysis on a power software application. The study also has certain limitations and issues of applying wavelet analysis like: i) it has poor directionality information, ii) very sensitive to data behind the analysis, iii) computationally intensive, iv) discretization (discrete wavelet transform is less efficient and natural), and v) it take lot of energy select correct wavelets for a specific purpose and to implement it correctly. Firstly, future research is expected to use more sophisticated analysis based on an extended dataset using advanced forms of wavelet analysis. Secondly future research is expected to use other nonlinear models to measure asymmetry. Thirdly other econometric techniques to attain rather comprehensive results. This study has significant contribution towards contemporary literature by way of: i) simultaneously revealing the cause-effect relationship in time-frequency space of CO_2 emissions by eleven different Asian countries without losing time-specific information, ii) extending the scarce literature on CO_2 emissions by eleven different Asian countries, iii) developing understanding of the effects of the phenomenon understudy, iv) providing better insights to environmental stakeholders, and v) providing new knowledge about transformation of patterns of emissions of CO_2 evolving through time and across different frequencies.

5. Conclusion

Assessment of CO_2 emissions by a country is importance because it has adverse effects on climate. It is the major contributor to global warming. Assessing the position of emerging economies as against its regional counter parts remained high in research agenda. This study is also aimed to assess position of Pakistan as against eminent Asian economies. Using the historical times series data of CO_2 emissions (kg per 2010 US$ of GDP) taken from WDI (Table 1) Pakistan's position is assessed using basic statistical techniques coupled with Wavelet analysis. The assessment of CO_2 emissions co-movement is considered a major debated issue in climate analysis so as to shed light, on the effective possible controls of CO_2 emissions at international level. The contemporary literature focuses on time-variation in co-movement but, in reality, CO_2 emissions have many time scales. In this study, using the wavelet coherence method, it is examined that how co-movement between eminent eleven Asian economies evolved over time and over time scale. The study provides evidence that there are time-variation and scale variation in co-movements between these economies. Results show a strong co-movement between CO_2 emissions by countries subject to study in the long run. The study found that there is no obvious lead–lag relationship between CO_2 emissions in these countries in short run. However, in the long run, Singapore and Japan, show leading effects on China, India, Indonesia, Iran, Korea, Malaysia, Pakistan, Sri Lanka and Turkey and this leading effect is gradually decreasing with the increasing time scale. This means that the Singapore and Japan still emits more CO_2 in Asia, but it is expected to be changed over time. The findings of the study have important implications for stakeholders. The strong co-movement at a longer time scale implies that stakeholders would not be able to reap simultaneously the benefits of expected CO_2 emission control. Our results show substantial variations in CO_2 emissions for both time and scale in international context. Results show that the stakeholders should rethink about the controls of CO_2 emissions. In this way this study contributed descriptive statistics about the phenomenon, time-series graphs, continuous wavelet power spectra, cross-wavelet coherency spectra and coherence plots spectra of CO_2 Emissions of aforementioned countries in contemporary literature.

References

Abban, O. J., Hongxing, Y., Nuta, A. C., Dankyi, A. B., Ofori, C., & Cobbinah, J. (2022). Renewable energy, economic growth, and CO2 emissions contained Co-movement in African oil-producing countries: A wavelet based analysis. *Energy Strategy Reviews*, *44*, 100977.

Abbasi, M. A., Parveen, S., Khan, S., & Kamal, M. A. (2020). Urbanization and energy consumption effects on carbon dioxide emissions: evidence from Asian-8 countries using panel data analysis. *Environmental Science and Pollution Research*, *27*(15), 18029-18043.

Adebayo, T. S., & Beton Kalmaz, D. (2021). Determinants of CO 2 emissions: empirical evidence from Egypt. *Environmental and Ecological Statistics*, *28*, 239-262.

Akbar, M., Hussain, A., Akbar, A., & Ullah, I. (2021). The dynamic association between healthcare spending, CO 2 emissions, and human development index in OECD countries: Evidence from panel VAR model. *Environment, development and sustainability*, *23*, 10470-10489.

Anwar, A., Younis, M., & Ullah, I. (2020). Impact of urbanization and economic growth on CO2 emission: a case of Far East Asian countries. *International Journal of Environmental Research and Public Health*, *17*(7), 2531.

Armah, M., Amewu, G., & Bossman, A. (2022). Time-frequency analysis of financial stress and global commodities prices: Insights from wavelet-based approaches. *Cogent Economics & Finance*, *10*(1), 2114161.

Audi, A. & Ali, A. (2023). The Role of Environmental Conditions and Purchasing Power Parity in Determining Quality of Life among Big Asian Cities. *International Journal of Energy Economics and Policy*, 13 (3), 292-305.

Audi, M. & Ali, A. (2023). Unveiling the Role of Business Freedom to Determine Environmental Degradation in Developing countries. *International Journal of Energy Economics and Policy*, 13(5), 157-164.

Azami, S., & Angazbani, F. (2020). CO2 response to business cycles: new evidence of the largest CO2-Emitting countries in Asia and the Middle East. *Journal of cleaner production*, *252*, 119743.

Batool, Z., Raza, S. M. F., Ali, S., & Abidin, S. Z. U. (2022). ICT, renewable energy, financial development, and CO2 emissions in developing countries of East and South Asia. *Environmental Science and Pollution Research*, *29*(23), 35025-35035.

Bloomfield, D. S., McAteer, R. J., Lites, B. W., Judge, P. G., Mathioudakis, M., & Keenan, F. P. (2004). Wavelet phase coherence analysis: application to a quiet-sun magnetic element. *The Astrophysical Journal*, *617*(1), 623.

Buschow, S., & Friederichs, P. (2020). Using wavelets to verify the scale structure of precipitation forecasts. *Advances in Statistical Climatology, Meteorology and Oceanography*, *6*(1), 13-30.

C. Dela Cruz, J., Ibera, J., M. Alcoy, J., & Erick R. Tulio, C. (2021, March). Deriving heart rate and respiratory rate from ecg using wavelet transform. In *2021 11th International Conference on Biomedical Engineering and Technology* (pp. 100-105).

Chapon, A., Ouarda, T. B., & Hamdi, Y. (2023). Imputation of missing values in environmental time series by D-vine copulas. *Weather and Climate Extremes*, 100591.

Das, D., & Manoharan, K. (2019). Emerging stock market co-movements in South Asia: wavelet approach. *International Journal of Managerial Finance*, *15*(2), 236-256.

Daud, S. N. S. S., & Sudirman, R. (2022). Wavelet based filters for artifact elimination in electroencephalography signal: A review. *Annals of Biomedical Engineering*, *50*(10), 1271-1291.

Donner, R., & Thiel, M. (2007). Scale-resolved phase coherence analysis of hemispheric sunspot activity: a new look at the north-south asymmetry. *Astronomy & Astrophysics*, *475*(3), L33-L36.

Erdogan, S., & Solarin, S. A. (2021). Stochastic convergence in carbon emissions based on a new Fourier-based wavelet unit root test. *Environmental Science and Pollution Research*, *28*(17), 21887-21899.

Gallegati, M. (2018). A systematic wavelet-based exploratory analysis of climatic variables. *Climatic Change*, *148*(1-2), 325-338.

Grinsted, A., Moore, J. C., & Jevrejeva, S. (2004). Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlinear processes in geophysics*, *11*(5/6), 561-566.

Hanif, I., Raza, S. M. F., Gago-de-Santos, P., & Abbas, Q. (2019). Fossil fuels, foreign direct investment, and economic growth have triggered CO2 emissions in emerging Asian economies: some empirical evidence. *Energy*, *171*, 493-501.

Hassan, M., Terrien, J., Karlsson, B., & Marque, C. (2010). Application of wavelet coherence to the detection of uterine electrical activity synchronization in labor. *Irbm*, *31*(3), 182-187.

Karp, A., & Van Vuuren, G. (2017). The capital asset pricing model and Fama-French three factor model in an emerging market environment. *International Business & Economics Research Journal (IBER)*, *16*(4), 231-256.

Kelly, B. C., Hughes, P. A., Aller, H. D., & Aller, M. F. (2003). The cross-wavelet transform and analysis of quasi-periodic behavior in the Pearson-readhead VLBI survey sources. *The Astrophysical Journal*, *591*(2), 695.

Khayyam, U., & Nazar, U. (2021). Energy production and CO2 emissions: The case of coal fired power plants under China Pakistan economic corridor. *Journal of Cleaner Production*, *281*, 124974.

Khoshnevis Yazdi, S., & Golestani Dariani, A. (2019). CO 2 emissions, urbanisation and economic growth: Evidence from Asian countries. *Economic research-Ekonomska istraživanja*, *32*(1), 510-530.

Li, K. J., Gao, P. X., & Zhan, L. S. (2009). Synchronization of hemispheric sunspot activity revisited: wavelet transform analyses. *The Astrophysical Journal*, *691*(1), 537.

Li, S., & Zhou, C. (2019). What are the impacts of demographic structure on CO2 emissions? A regional analysis in China via heterogeneous panel estimates. *Science of the Total Environment*, *650*, 2021-2031.

Liu, L. T., Hsu, H. T., & Grafarend, E. W. (2005). Wavelet coherence analysis of length-of-day variations and El Nino-southern oscillation. *Journal of Geodynamics*, *39*(3), 267-275.

Luo, R., Li, Y., Wang, Z., & Sun, M. (2022). Co-movement between carbon prices and energy prices in time and frequency domains: a wavelet-based analysis for Beijing carbon emission trading system. *International Journal of Environmental Research and Public Health*, *19*(9), 5217.

Mutascu, M., & Sokic, A. (2020). Trade openness-CO2 emissions nexus: a wavelet evidence from EU. *Environmental Modeling & Assessment*, *25*(3), 411-428.

Neves, S. A., Marques, A. C., & Patrício, M. (2020). Determinants of CO2 emissions in European Union countries: does environmental regulation reduce environmental pollution?. *Economic Analysis and Policy*, *68*, 114-125.

Njoh, A. J. (2021). Renewable energy as a determinant of inter-country differentials in CO2 emissions in Africa. *Renewable Energy*, *172*, 1225-1232.

Nkalu, C. N., & Edeme, R. K. (2019). Environmental hazards and life expectancy in Africa: evidence from GARCH model. *Sage Open*, *9*(1), 2158244019830500.

Parker, S., & Bhatti, M. I. (2020). Dynamics and drivers of per capita CO2 emissions in Asia. *Energy Economics*, *89*, 104798.

Qassim, Y. T., Cutmore, T. R., James, D. A., & Rowlands, D. D. (2013). Wavelet coherence of EEG signals for a visual oddball task. *Computers in Biology and Medicine*, *43*(1), 23-31.

Qazi, T. F., Niazi, A. A. K., & Ahmad, M. S. (2022). Co-Movement Between Pakistan Stock Exchange Index, Gold Price and Foreign Exchange Rate: A Wavelet Analysis. International Conference on Banking, Insurance & Business Management 2022 (CBIBM 2022), 558–579.

Rahman, M. M., & Alam, K. (2022). CO2 Emissions in Asia–Pacific region: do energy use, economic growth, financial development, and international trade have detrimental effects?. *Sustainability*, *14*(9), 5420.

Rhif, M., Ben Abbes, A., Farah, I. R., Martínez, B., & Sang, Y. (2019). Wavelet transform application for/in non-stationary time-series analysis: A review. *Applied Sciences*, *9*(7), 1345.

Salahuddin, M., Ali, M. I., Vink, N., & Gow, J. (2019). The effects of urbanization and globalization on CO 2 emissions: evidence from the Sub-Saharan Africa (SSA) countries. *Environmental Science and Pollution Research*, *26*, 2699-2709.

Scholkmann, F., & Dommer, L. (2012). Between-brain coherence during joint n-back task performance: A two-person functional near-infrared spectroscopy... *Behavioural Brain Research*, *234(2)*, 212-222.

Sezer, O. B., Gudelek, M. U., & Ozbayoglu, A. M. (2020). Financial time series forecasting with deep learning: A systematic literature review: 2005–2019. *Applied soft computing*, *90*, 106181.

Shahbaz, M., Loganathan, N., Muzaffar, A. T., Ahmed, K., & Jabran, M. A. (2016). How urbanization affects CO2 emissions in Malaysia? The application of STIRPAT model. *Renewable and Sustainable Energy Reviews*, *57*, 83-93.

Soni, R. K., Nandan, T., & Chatnani, N. N. (2023). Dynamic association of economic policy uncertainty with oil, stock and gold: a wavelet-based approach. *Journal of Economic Studies*.

Wirsing, K. (2020). Time frequency analysis of wavelet and Fourier transform. *Wavelet theory*.

Wu, J., Abban, O. J., Boadi, A. D., Addae, E. A., Akhtar, M., Hongxing, Y., & Ofori, C. (2022). Time–frequency contained co-movement of renewable electricity production, globalization, and co2 emissions: A wavelet-based analysis in Asia. *Energy Reports*, *8*, 15189-15205.

Younis, I., Longsheng, C., Basheer, M. F., & Joyo, A. S. (2020). Stock market comovements among Asian emerging economies: A wavelet-based approach. *Plos one*, *15*(10), e0240472.

Yu, Z., Wang, Z., & Wang, J. (2023). Continuous Wavelet Transform and Dynamic Time Warping-Based Fine Division and Correlation of Glutenite Sedimentary Cycles. *Mathematical Geosciences*, *55*(4), 521-539.

Zamani, A., Samiee, J., & Kirby, J. F. (2013). Estimating the mechanical anisotropy of the Iranian lithosphere using the wavelet coherence method. *Tectonophysics*, *601*, 139-147.

Zeng, S., Liu, Y., Ding, J., & Xu, D. (2020). An empirical analysis of energy consumption, FDI and high quality development based on time series data of Zhejiang province. *International journal of environmental research and public health*, *17*(9), 3321.

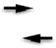
Zhu, L., Wang, Y., & Fan, Q. (2014). MODWT-ARMA model for time series prediction. *Applied Mathematical Modelling*, *38*(5-6), 1859-1865.

Ziaei, S. M. (2015). Effects of financial development indicators on energy consumption and CO2 emission of European, East Asian and Oceania countries. *Renewable and Sustainable Energy Reviews*, *42*, 752-759.

Supporting information: Supporting information is given as Annexure-I and Annexure-II below.

Acknowledgments: Our colleagues for helping us in data analysis. We also wish to thank anonymous referees for valuable comments and suggestions.

Annexure I
Formulae

Variable	:	Purpose
u	:	Represents the position of the respective wavelet
t	:	Time
s	:	Represents the parameter for dilation of scale (defines how the wavelet is scaled, i.e., stretched or compressed).
$\psi(.)$	:	Real-valued or a complex-valued function $\psi(.)$ defined over the real axis.
e	:	Exponential function
s	:	s represents the dilation or scale factor (defines how the wavelet is scaled i.e. stretched or compressed)
i	:	Imaginary unit
ω_0	:	Wavelet's central frequency
M	:	Morlet wavelet
l	:	Length
π	:	It is used to represent a usual mathematical constant
$\int_{-\infty}^{\infty} dt$	:	In mathematics, an integral assigns numbers to functions in a way that describes displacement, area, volume, and other concepts that arise by combining infinitesimal data. The process of finding integrals is called integration. Or Area under the curve
c_ψ	:	Reconstruction of a time series from its continuous wavelet transform
$W_x()$	:	Can be found through a $\psi(.)$ projection on the time-series,
ψ	:	Mother wavelet function
$\psi_{u,s}$	:	Daughter wavelet function
d_s	:	***ds*** is differential where ***ds*** represents a small change in ***s***
$D()$	:	Corresponding distribution for the local wavelet power spectrum
σ	:	Standard deviations
p	:	ρ denotes the probability for a *pdf* (probability density function)
W_n^x	:	W is wavelet transforms *n* is the time index, x means time series *x*
*	:	Indicates complex conjugate
P_f	:	P_f is the mean of the spectrum at the Fourier frequency *f* that corresponds to the wavelet
$\chi 2$	:	Chi Square
v	:	Takes the values of 1 or 2 for real or complex wavelets.
n	:	Number of observation
X	:	Time series X-axis
Y	:	Time series Y-axis
σ_X	:	Denotes the standard deviations of *x*
σ_Y	:	Denotes the standard deviations of *y*
$Z_v(p)$	:	Represents the confidence interval. where p denotes the probability for a *pdf* (probability density function)
$P_{\mathcal{K}}^X$ and $P_{\mathcal{K}}^Y$	:	The theoretical distribution of the cross-wavelet power of two signals with power spectra $P_{\mathcal{K}}^X$ and $P_{\mathcal{K}}^Y$.
$R^2(u,s)$	:	Wavelet coherence
S	:	The smoothing parameter
W_{xy}	:	Power spectrum for cross wavelet
W_x	:	Power spectrum for wavelet time series x
W_y	:	Power spectrum for wavelet time series y
$\langle\cdot\rangle$	:	$\langle\cdot\rangle$ indicates smoothing in both time and scale
Φ	:	Corresponding phase at the position x and the wavenumber K with $\Phi(x) = \phi(x, K(x))$ (rad)
$\phi_{x,y}$	:	Phase difference of two time series variables
$\tan^{-1}$	:	Trigonometric function
$\mathfrak{I}$	:	Imaginary
$\mathcal{R}$	:	The smoothed real
	:	Positively correlated
	:	Negatively correlated
	:	x is leading, positively correlated
	:	x is lagging, positively correlated
	:	x is lagging, negatively correlated
	:	x is leading, negatively correlated
$x(t) \in L^2(\mathbb{R})$	:	Function is called an orthonormal wavelet

Annexure II
Definitions of Different Terms Used

Term/Construct	Definition
Amplitude	Amplitude, in physics, the maximum displacement or distance moved by a point on a vibrating body or wave measured from its equilibrium position. It is equal to one-half the length of the vibration path.
Complex Conjugate	In mathematics, the complex conjugate of a complex number is the number with an equal real part and an imaginary part equal in magnitude but opposite in sign.
Concentration	Divide the mass of the solute by the total volume of the solution. Write out the equation C = m/V, where m is the mass of the solute and V is the total volume of the solution. Plug in the values you found for the mass and volume, and divide them to find the concentration of your solution.
Cone of Influence (COI)	The cone of influence (COI) is the region where the wavelet power spectra are distorted because of the influence of the end points of finite length signals. Based on the traditional methods, it is difficult to extract intrinsic feature in these region of the wavelet power spectra of climatic signals.
Contour plot	A contour plot is a graphical technique for representing a 3-dimensional surface by plotting constant *z* slices, called contours, on a 2-dimensional format. That is, given a value for *z*, lines are drawn for connecting the (x, y) coordinates where that *z* value occurs.
Convolution	In mathematics (in particular, functional analysis), convolution is a mathematical operation on two functions (f and g) that produces a third function ( ) that expresses how the shape of one is modified by the other. The term convolution refers to both the result function and to the process of computing it.
Dilation Meaning in Math	Dilation is a transformation, which is used to resize the object. Dilation is used to make the objects larger or smaller. This transformation produces an image that is the same as the original shape.
Integral	A function of which a given function is the derivative, i.e. which yields that function when differentiated, and which may express the area under the curve of a graph of the function. In mathematics, an integral assigns numbers to functions in a way that describes displacement, area, volume, and other concepts that arise by combining infinitesimal data. The process of finding integrals is called integration.
Multi-resolution	A computational tool, multiresolution can be applied to a variety of problems in signal and image processing. For instance, feature detection and extraction can be performed quickly and efficiently using a multi-resolutional method to analyze images.
Multi-resolution Analysis	Classically the term multi-resolution has been intimately connected with the study of wavelets. Wavelets are useful to describe mathematical objects such as functions (or signals) at different levels of resolution. For example, an image can be described at different levels of resolution
Oscillation:	Movement back and forth in a regular rhythm. Or Regular variation in magnitude or position about a central point, especially of an electric current or voltage
Period	A length or portion of time
Phase	A distinct period or stage in a series of events or a process of change or development.
Probability density function:	Probability density function (PDF) is a statistical expression that defines a probability distribution (the likelihood of an outcome) for a discrete random variable (e.g., a stock or ETF) as opposed to a continuous random variable.
Randomize	Make (a set of items, people, etc.) unpredictable, unsystematic, or random in order or arrangement. Randomization in an experiment refers to a random assignment of participants to the treatment in an experiment. OR, for instance we can say that randomization is assignment of treatment to the participants randomly. For example: a teacher decides to take a viva in the class and randomly starts asking the students.
Resolution exactly	Resolution measures the number of pixels in digital image or display.
Scaling	Scaling an object means multiplying every linear dimension of it by the same factor. Thus you change the size of the object, but not its shape.
Seismograph	An instrument that measures and records details of earthquakes, such as force and duration.
Shifting	A transformation in which a graph or geometric figure is picked up and moved to another location without any change in size or orientation. One kind of transformation involves shifting the entire graph of a function up, down, right, or left.
Smoothing	In smoothing, the data points of a signal are modified so individual points higher than the adjacent points (presumably because of noise) are reduced, and points that are lower than the adjacent points are increased leading to a smoother signal. ... Many different algorithms are used in smoothing.
Spectrum	A band of colors, as seen in a rainbow, produced by separation of the components of light by their different degrees of refraction according to wavelength. Used to classify something in terms of its position on a scale between two extreme points
Surrogate	A substitute
Time	The indefinite continued progress of existence and events in the past, present, and future regarded as a whole
Transform	To change completely the appearance or character of something or someone, especially so that that thing or person is improved
Translation	Translation means the displacement of a figure or a shape from one place to another. In translation, a figure can move upward, downward, right, left or anywhere in the coordinate system. A translation is a transformation that moves every point in a figure the same distance in the same direction.
Vector:	A vector is an object that has both a magnitude and a direction. Geometrically, we can picture a vector as a directed line segment, whose length is the magnitude of the vector and with an arrow indicating the direction. The direction of the vector is from its tail to its head.
Wavelet Squared Coherence	Description. wcoh = wcoherence (x, y) returns the magnitude-squared wavelet coherence, which is a measure of the correlation between signals x and y in the time-frequency plane. Wavelet coherence is useful for analyzing non-stationary signals. The inputs x and y must be equal length, 1-D, real-valued signals.
Wavelet transform as a convolution	A wavelet transform is essentially a convolution with a bunch of functions chosen to be "compact" in frequency and time.
Wavelets and multi-resolution processing	Wavelet transform is used to analyze a signal (image) into different frequency components at different resolution scales (i.e. multi-resolution). This allows revealing image's spatial and frequency attributes simultaneously. In addition, features that might go undetected at one resolution may be easy to spot at another. Multi-resolution theory incorporates image pyramid and sub-band coding techniques.